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The total demand scale: a new measure of quality for static and dynamic origin–destination trip tables

Michel Bierlaire

Department of Mathematics, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland

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Abstract

We propose a measure of quality for OD trip tables estimated from link counts. This measure, called the *total demand scale* is based only on the estimated OD trip table and on the assignment matrix used to estimate it. It is independent of the OD estimation method and of the technique used to overcome the underdetermination of the OD estimation problem. The total demand scale measures the intrinsic underdetermination related to the OD estimation problem, considering only the network topology and the underlying route choice model. Because it identifies the level of arbitrariness introduced by the a priori matrix in the estimation process, our approach helps to evaluate if an important amount of resources must be invested for the a priori matrix, or if the available link counts contains sufficient information. The computation of the total demand scale enables to identify at zero additional cost those OD pairs such that no measurement is available on any path linking them. A MATLAB code for the computation of the total demand scale is also provided. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The problem of estimating origin–destination (OD) trip tables [1] from flows observations on selected links of the transportation network has been widely studied in the literature (see namely Bell, 1983; Cascetta, 1984; Van Zuylen and Willumsen, 1980 among the most cited papers). Trip tables are used as inputs to many models. Consequently, errors in OD tables have repercussions on subsequent stages of transportation analyses. It is therefore important to compute some

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E-mail address: michel.bierlaire@epfl.ch (M. Bierlaire).

[1] In practice, they are often called origin–destination *matrices*. In this paper, they are represented by a vector, and not by a matrix. Therefore, we prefer the name *trip table* to avoid any confusion.

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measures of uncertainty for the estimated trip tables. Typically, statistical measures derived from the variance and covariance of the input data of the estimation process are used for that purpose (Cascetta, 1984; Hazelton, 2000). These statistical measures play an important role, but are not sufficient. Indeed, due to the intrinsic overdetermination of the OD estimation problem, arbitrary decisions must be made within the estimation process. In this paper, we propose a measure of quality that determines how arbitrary such modeling decisions can be. We emphasize that it should be used in addition to, not instead of, standard statistical techniques.

The rest of this paper is organized as follows. The static and dynamic OD estimation (DODE) problems are first presented in Sections 2 and 3, respectively. The problem of overdetermination is described in Section 4 and the new measure of quality is introduced in Section 5. The case where some OD pairs are not captured by any link observation is treated in Section 6. Finally, some sensitivity analysis of the total demand scale (TDS) is performed in Section 7.

2. Static OD estimation

The static OD estimation (SODE) problem is the inverse problem of the static traffic assignment (STA) problem. The STA problem consists of finding link flows given an OD trip table. Several approaches, based on specific assumptions about route choice and about the impact of congestion on route choice have been proposed. We refer the reader to Sheffi (1985) and Patriksson (1994) for a review of existing traffic assignment models. The SODE problem can be formulated as follows.

Let $G = (V, E)$ be a directed graph representing a transportation network. V is the set of vertices. E is the set of n edges, or links. Let $\Omega \subseteq V \times V$ be the set of m OD pairs in the network, and $\hat{E} \subseteq E$ be the set of r links where flow observations are available.

If $q \in \mathbb{R}^m$ represents the origin–destination flows for each OD pair in Ω, the vector $v = A(q) \in \mathbb{R}^n$ represents the link flows for each link in E, as provided by a given assignment model. The vector $v_{\hat{E}}(q) \in \mathbb{R}^r$ contains only entries of v corresponding to links in $\hat{E}$.

Given a vector of observed link flows $\hat{v} \in \mathbb{R}^r$, the OD estimation problem consists of finding an OD trip table q such that $v_{\hat{E}}(q)$ is as close as possible to $\hat{v}$. Existing approaches capture that concept of closeness through least squares (Cascetta, 1984; Ashok and Ben-Akiva, 1993; Bierlaire and Toint, 1995), entropy (Van Zuylen and Willumsen, 1980; Bell, 1984) and maximum likelihood (Spiess, 1987), among others.

The assignment function $A(q)$ plays a key role in this process. The assignment process can be decomposed as follows, independently of the nature of the model (all-or-nothing, equilibrium or any other).

For all $i \in \Omega$, let $\mathscr{P}_i$ be the set of p_i paths linking OD pair i. We assume that the network is connected, and that $\mathscr{P}_i \neq \emptyset\ \forall i$. We have that $p = \sum_{i \in \Omega} p_i$ is the total number of paths in the network.

We define the matrix $L \in \mathbb{R}^{n \times p}$ as follows:

$$L_{k\ell} = \begin{cases} 1 & \text{if link } k \text{ belongs to path } \ell, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

This matrix represents the link-path incidence matrix, and depends only on the network topology.

We also define the route choice matrix $C \in \mathbb{R}^{p\times m}$, where $C_{\ell i}$ is the proportion of trips from OD pair i using path ℓ. To be consistent with route choice models (see for example Ben-Akiva and Bierlaire, 1999), we assume that

$$\sum_{\ell\in\mathscr{P}_i} C_{\ell i} = 1 \quad \forall i \in \Omega \tag{2}$$

and

$$C_{\ell i} = 0 \quad \text{if } \ell \notin \mathscr{P}_i. \tag{3}$$

If we denote by $\mathscr{I}(k)$ the (unique) OD pair connected via path k, we obviously have $C_{kj} = 0$ if $j \neq \mathscr{I}(k)$.

Clearly, in congested networks, the matrix C depends on the prevailing traffic conditions. Unfortunately, incorporating that dependence into the model significantly complicates the OD estimation problem. Therefore, most approaches assume that C is given for the OD estimation, and iterate between OD estimation and traffic assignment until some sort of convergence is reached. It is namely the approach suggested by the SATURN system (Van Vliet, 1982). Recent techniques, based on bilevel optimization problems, include explicitly traffic equilibrium conditions in the model (Barceló and Casas, 1999). In this paper, we only consider the matrix C which is available at the end of the estimation process, ignoring the underlying modeling approach and all possible modifications of C during the estimation itself.

Defining the *assignment matrix* $A \in \mathbb{R}^{n\times m}$

$$A = LC, \tag{4}$$

the assignment model is given by

$$A(q) = Aq. \tag{5}$$

Therefore, we can write the OD estimation problem as follows:

$$q^* = \operatorname*{argmin}_q d(\hat{A}q, \hat{v}), \tag{6}$$

where $\hat{A}$ is the $r \times m$ matrix composed of the rows of A corresponding to $\hat{E}$, and d is any suitable distance function.

3. Dynamic OD estimation

The DODE problem is the inverse problem of the dynamic traffic assignment (DTA) problem (Cremer and Keller, 1987; Nihan and Davis, 1987; Sherali et al., 1997; Li and De Moor, 1999). Given a discretization of the period under consideration into T time intervals $t_1, \ldots, t_T$, and given T origin–destination trip tables q_i, the DTA problem consists of finding time-dependent link flows $v_i = A(q)$, $i = 1, \ldots, T$. It can be solved analytically (Cascetta and Cantarella, 1990; Friesz et al., 1993) or by simulation tools (Mahmassani et al., 1993; Yang and Koutsopoulos, 1997; Ben-Akiva et al., forthcoming).

Let $G = (V, E)$ be a directed graph representing a transportation network, and $\bigcup_{i=1}^{T} t_T$ the time horizon discretized into T time intervals. As in the static case, V is the set of vertices and E is the set of n links. Let $\Omega \subseteq V \times V$ be the set of m OD pairs in the network, and $\hat{E} \subseteq E$ be the set of r links where flow observations are available. We also consider D, a set of time-dependent link travel times. $D_{\ell i}$ is the number of time intervals needed to go through link ℓ when vehicles arrive during time interval t_i. We denote by $\bar{G}(V \times T, \bar{E})$ the time-space expansion of G based on D. In $\bar{G}$, there is a link between node (ℓ, i) and (m, j) if there is a link in G between nodes ℓ and m and if $t_{i+D_{\ell i}} = t_j$. Using this representation, the DODE problem is equivalent to an SODE problem in $\bar{G}$. Consequently, we focus on the SODE problem in the rest of this paper.

4. Overdetermination

For most practical applications, the SODE problem is overdetermined. That is, there is an infinite number of valid trip tables that, when assigned on the network, produce link flows that are at the same optimal distance from the observations. For example, if the distance d is captured by least squares, problem (6) is equivalent to solving the normal equations

$$\hat{A}^{\mathrm{T}} \hat{A} q = \hat{A}^{\mathrm{T}} \hat{v}. \tag{7}$$

This is a linear system of r equations with m unknowns. And the number of OD pairs m is usually much greater than the number of observed link counts r.

To overcome that problem, it is common to use an a priori OD trip table q_0, called *a priori trip table* and to select among the infinite number of potential candidates the one that is closest to the a priori table. The problem has then a unique solution. The a priori table is usually obtained from a previous study, or from road side interviews. Sometimes, when no data are available, the a priori table is filled with arbitrary values, typically all 1s or all 0s. Other techniques, based on additional data to identify a structure of the matrix, have also been proposed to reduce that underdetermination (see, for example, Bierlaire and Toint, 1995).

We propose here a measure of the level of arbitrariness of the a priori OD table. The measure is independent of the data and of the estimation method. It is based on a structural analysis of the model, and is complementary to statistical measures, such as confidence interval. This measure depends only on the estimated trip table q^* and on the assignment matrix (4). It does not depend on the a priori trip table, neither on the observed link flows $\hat{v}$. Also, it is independent of the OD estimation method.

5. The total demand scale

We assume that an OD estimation problem has been solved based on flow data from links in $\hat{E}$, using any appropriate algorithm. We assume that the algorithm was able to capture the intrinsic errors in the data and the possible inconsistencies between the observed counts and the route choice assumptions. Also, even if the route choice matrix C is not unique by nature, the estimated OD table produced by the estimation method was eventually based on a single route choice matrix C and, therefore, on a single assignment matrix A. The estimated OD table is denoted by q^* and the assignment matrix is $A = LC$.

Let $v^* = Aq^*$ be the link flows obtained when q^* is assigned on the network. The vector $v^*_{\hat{E}}$ contains only the entries of v^* corresponding to links in $\hat{E}$. $\hat{A}$ is the $r \times m$ matrix composed of the rows of A corresponding to $\hat{E}$.

If we ignore the a priori trip table, any OD trip table q satisfying

$$\hat{A}q = v^*_{\hat{E}} \tag{8}$$

and

$$q \geqslant 0 \tag{9}$$

could also have been considered to be a good estimation of the "true" trip table. We derive the quality measure of q^* by characterizing the set of all tables q verifying (8) and (9). The larger this set, the more arbitrary the role of the a priori matrix.

By writing $q = q^* + \delta$, it appears that q verifies (8) if and only if δ is in $\ker(\hat{A})$, the null space of $\hat{A}$. The dimension of that subspace is $m - \operatorname{rank}(\hat{A})$. This number provides a first measure of the size of the underdetermination. Unfortunately, it does not take into account (9) and, therefore, may provide a pessimistic measure of quality.

To obtain a more accurate measure of quality, we must consider (8) and (9) together. We observe that the set of q verifying both equations is a polyhedron. Therefore, a good measure of the underdetermination would be the volume of that polyhedron. Such a measure makes sense, as the polyhedron is bounded if measurements are available from each link in the network. Therefore, an occurrence of an unbounded polyhedron may only be due to lack of data (see Section 6). This result is proven by the following theorem.

Theorem 1. *If $\hat{E} = E$, and if the network is connected, the polyhedron $\mathcal{Q}$ defined by* (8) *and* (9) *is bounded.*

Proof. If $\hat{E} = E$, we have $\hat{A} = A$ and $v^*_{\hat{E}} = v^*$. By definition, the polyhedron is not empty, because it contains at least q^*.

Let d be any vector in $\ker(A)$. We have, for any link i,

$$0 = \sum_j A_{ij} d_j = \sum_j \sum_k L_{ik} C_{kj} d_j = \sum_k L_{ik} \sum_j C_{kj} d_j = \sum_k L_{ik} C_{k\mathscr{I}(k)} d_{\mathscr{I}(k)}, \tag{10}$$

where $\mathscr{I}(k)$ is the (unique) OD pair linked by path k.

Suppose that the polyhedron is not bounded. Therefore, there is a direction $d^* \in \mathbb{R}^m$, $d^* \geqslant 0$, $d^* \neq 0$, such that

$$q^* + \alpha d^* \in \mathcal{Q} \quad \forall \alpha \geqslant 0. \tag{11}$$

We assume, without loss of generality, that $d^*_1 > 0$. Because the graph is connected, there is at least a path linking OD pair 1. Therefore, there is at least one k such that $C_{k\mathscr{I}(k)} = C_{k1} \neq 0$, and at least one link i on that path, so that $L_{ik} = 1$.

We obtain a contradiction, as $d^* \in \ker(A)$ but does not verify (10) for all links i belonging to path k. □

Even if some practical techniques have been proposed (Bueler et al., 1998), the problem of computing the volume of a polyhedron is known to be hard (Dyer and Frieze, 1988). In the context of OD estimation, the polyhedron dimension can be large, especially in the dynamic case where the time-space expansion of the network is considered.

The Total Demand Scale (TDS) is a measure considering both (8) and (9) (contrarily to the measure based on the null space of $\hat{A}$) and easy to compute (contrarily to the volume of the polyhedron). If we define

$$\begin{aligned} \phi_{\min} = \min_q \quad & q^{\mathrm{T}} e, \\ \text{s.c.} \quad & \hat{A} q = v^*_{\hat{E}}, \\ & q \geqslant 0, \end{aligned} \tag{12}$$

where $e \in \mathbb{R}^m$ is a vector composed only of ones, and

$$\begin{aligned} \phi_{\max} = \max_q \quad & q^{\mathrm{T}} e, \\ \text{s.c.} \quad & \hat{A} q = v^*_{\hat{E}}, \\ & q \geqslant 0, \end{aligned} \tag{13}$$

the TDS $\Phi(q^*, \hat{A})$ is defined by

$$\Phi(q^*, \hat{A}) = \phi_{\max} - \phi_{\min}, \tag{14}$$

where $\phi_{\min}$ and $\phi_{\max}$ are the minimum and the maximum (resp.) levels of total demand consistent with the constraints (8) and (9).

The interval $[\phi_{\min}, \phi_{\max}]$ is the range, or scale, of the total level of demand in the network. If the interval is empty, that is if $\phi_{\min} = \phi_{\max}$ or if $\Phi(q^*, \hat{A}) = 0$, it means that all OD tables q verifying (8) and (9) have the exact same total demand as q^*. Therefore, the error due to the problem underdetermination may affect only the way the total demand is spread over OD pairs.

5.1. Examples

We consider the network represented at Fig. 1, with nine OD pairs (see Table 1).

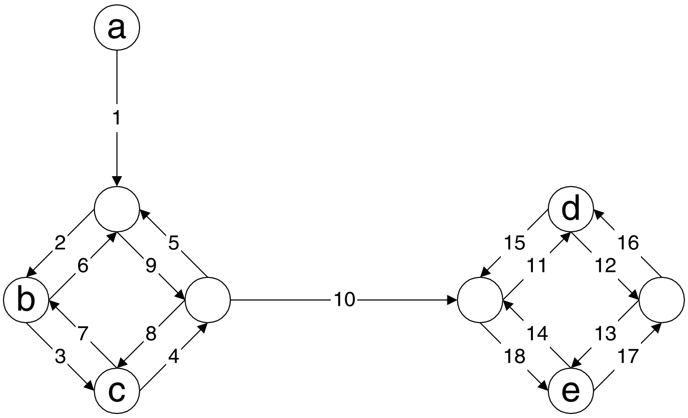

Fig. 1. Network example.

Table 1
OD pairs for the example

#	Pair	#	Pair	#	Pair
1	(a, d)	4	(b, e)	7	(a, c)
2	(a, e)	5	(c, d)	8	(a, b)
3	(b, d)	6	(c, e)	9	(d, e)

We assume equal probability among available paths between each OD. Note that OD pairs 1–6 have 4 loopless paths each, while OD pairs 7–9 have only two such paths, for a total of 30 paths.

We consider first an instance of the OD estimation problem based on $\hat{E} = \{9, 13\}$ and $v^*_{\hat{E}} = (400\ 200)^{\mathrm{T}}$. The corresponding assignment matrix $\hat{A}$ is

$$\begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 & 0.5 & 0 & 0 & 0.5 \end{pmatrix}. \tag{15}$$

The solution to (12) is $\phi_{\min} = 800$, obtained for example with

$$q_{\min} = \begin{pmatrix} 80 & \frac{400}{3} & 80 & \frac{400}{3} & 80 & \frac{400}{3} & 80 & 80 & 0 \end{pmatrix}^{\mathrm{T}}. \tag{16}$$

The solution to (13) is $\phi_{\max} = 1200$, obtained for example with

$$q_{\max} = \begin{pmatrix} 160 & 0 & 160 & 0 & 160 & 0 & 160 & 160 & 400 \end{pmatrix}^{\mathrm{T}}. \tag{17}$$

Therefore, the TDS for this example is 400.

We consider another instance of the OD estimation problem based on $\hat{E} = \{9, 15\}$ and $v^*_{\hat{E}} = (400\ 50)^{\mathrm{T}}$. The corresponding assignment matrix $\hat{A}$ is

$$\begin{pmatrix} 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \end{pmatrix}. \tag{18}$$

Here, both $\phi_{\min}$ and $\phi_{\max}$ are equal to 900. For example, the OD trip table q_{100} where all entries are equal to 100 veh/h is a possible solution to the problem. Actually, any OD table

$$q = \begin{pmatrix} 100 - \sum_{i=1}^{7} \alpha_i \\ 100 + \alpha_1 \\ 100 + \alpha_2 \\ 100 + \alpha_3 \\ 100 + \alpha_4 \\ 100 + \alpha_5 \\ 100 + \alpha_6 \\ 100 + \alpha_7 \\ 100 \end{pmatrix} \tag{19}$$

for any α_i, $i = 1, \ldots, 7$ such that $q \geqslant 0$ is a solution to the problem. We note that each OD table q verifying (19) can be obtained from q_{100} by transferring demand from OD pairs 2–8 to OD pair 1, or the other way around, while keeping the OD table positive. Therefore, the total level of demand remains unchanged within this set of OD tables. This illustrates that a TDS being zero does not necessarily mean a unique solution to the OD estimation problem.

Note that installing a counting device on link 10 that is, $\hat{E} = (10)$, produces also a TDS of 0. This result is intuitive because the overall demand has to go through that link, as the example is designed. Again, it illustrates that a TDS of zero does not correspond to an absence of arbitrariness in the OD estimation process.

Finally, we consider an instance of the OD estimation problem based on $\hat{E} = \{1, 6\}$ and $v^*_{\hat{E}} = (400\ 200)^{\mathrm{T}}$. The corresponding assignment matrix $\hat{A}$ is

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0 \end{pmatrix}. \tag{20}$$

The value of $\phi_{\min}$ is 800, achieved for example with a flow of 400 for OD pair (a, d) and a flow of 400 for OD pair (b, d). The value $\phi_{\max}$ is $+\infty$. Indeed, problem (13) is not bounded with this example. No path linking OD pair (d, e) uses links 1 or 6 where data are available. Therefore, the OD flow for this OD pair can be any positive number.

These three examples illustrate how useful the TDS is to appreciate the quality of an estimated OD table.

1. If TDS = 0, it means that the OD estimation algorithm has captured correctly the total level of demand in the network. The underdetermination relates only to the repartition of that demand across OD pairs.
2. If TDS > 0, we have a measure of the range of possible values for the total demand. The underdetermination is twofold: the total demand and the demand repartition.
3. If TDS = $+\infty$, it means that some OD pairs are not captured at all by the link flow data.

From a practical viewpoint, a larger TDS requires a better a priori matrix. The TDS helps to assess the level of investment necessary to collect data and build the a priori matrix.

The case where TDS is infinite is by far the most problematic, and may arise often in practice. In that case, TDS does not provide information at all, and a deeper quality analysis is desirable in that context. Such an analysis is proposed in the following section.

6. Dealing with unboundedness

The case where TDS is infinite appears when the polyhedron defined by (8) and (9) is not bounded. In this case, some OD pairs can take arbitrarily large values. After the problem has been detected through the TDS computation, two questions arise: what are the OD pairs causing the unboundedness and how good is the quality of the remaining OD pairs?

6.1. Detecting unbounded OD pairs

Unbounded OD pairs can be easily detected if the simplex algorithm (Dantzig, 1963) is used to solve the linear program (13) (problem (12) is always bounded). Indeed, if the linear program is not bounded, it can be shown (Bertsimas and Tsitsiklis, 1997) that the simplex algorithm finds a vector $d \geqslant 0$ such that

$$\hat{A}d = 0 \tag{21}$$

and

$$d^{\mathrm{T}}e > 0. \tag{22}$$

All non-zero entries of this vector d correspond to unbounded OD pairs.

6.2. Measuring the quality of the remaining OD flows

At this point, we are able to identify OD pairs for which available link flow measurements contain no information. We define now a measure of quality for the remaining OD flows estimates.

Suppose that we have applied the simplex method to solve (13), and that we have detected a direction $d \geqslant 0$ verifying (21) and (22). As we are not interested anymore in that direction, we restrict our analysis to the subspace orthogonal to d. We replace (12) by

$$\begin{aligned} \phi_{\min} = \max_{q} \quad & q^{\mathrm{T}}e \\ \text{s.c.} \quad & \hat{A}q = v^*_{\hat{E}}, \\ & d^{\mathrm{T}}q = 0, \\ & q \geqslant 0, \end{aligned} \tag{23}$$

and (13) by

$$\begin{aligned} \phi_{\max} = \max_{q} \quad & q^{\mathrm{T}}e \\ \text{s.c.} \quad & \hat{A}q = v^*_{\hat{E}}, \\ & d^{\mathrm{T}}q = 0, \\ & q \geqslant 0. \end{aligned} \tag{24}$$

If (24) is also unbounded, we start the process again. Considering the last example of Section 5.1, based on assignment matrix (20), we have that $\phi_{\min} = \phi_{\max} = 800$ and, therefore, the TDS is zero. The MATLAB code to compute the TDS is reported in Fig. 2. It is based on the `SIMPLEX` function, described in Fig. 3. In that code, $\phi_{\min}$ is `DMIN` and $\phi_{\max}$ is `DMAX`. The OD trip tables solution to linear programs (12) and (13) (or (23) and (24) if applicable) are `QMIN` and `QMAX`, respectively. Finally, the set of directions of unboundedness are stored in `DIR`.

7. Sensitivity

As mentioned above, the TDS provides a measure of quality of an estimated OD table, based on the network topology and the route choice assumptions. Clearly, assumptions related to the route choice model are usually more arbitrary than the network topology. The TDS is sensitive to the quality of the route choice model. We illustrate this with the example from Section 5.1 and the following route choice assumption. A probability of $\varepsilon/2$ is assigned to paths listed in Table 2. A probability of $(1-\varepsilon)/2$ is assigned to paths listed in Table 3. A probability of ε is assigned to paths listed in the first column of Table 4. The paths in the second column are assigned a

```
function [DMIN,DMAX,QMIN,QMAX,DIR] = TDS(A,qstar)
%function [DMIN,DMAX,QMIN,QMAX,DIR] = TDS(A,qstar)
%Michel Bierlaire EPFL Feb 2000
%This function computes the total demand scale
%for the estimated OD table qstar and the assignment matrix A.

v = A*qstar ;
[n,m] = size(A)
e = ones(m,1);

D = [] ;

[value_opt,primal_var,table_opt,dir,exitflag] = SIMPLEX(A,v,-e);

%  exitflag: 0 if an optimum solution is found
%            1 if the problem is not bounded
%            2 if the problem is infeasible
while exitflag == 1
   D = [D ; dir'] ;
   A = [A ; D] ;
   v = [v ; zeros(size(D,1),1)] ;
   [value_opt,primal_var,table_opt,dir,exitflag] = SIMPLEX(A,v,-e);
end
DMAX = -value_opt ;
QMAX = primal_var ;

[value_opt,primal_var,table_opt,dir,exitflag] = SIMPLEX(A,v,e) ;

DMIN = value_opt ;
QMIN = primal_var ;
DIR = D ;
return
```

Fig. 2. MATLAB code for the total demand scale.

```
function [value_opt,primal_var,table_opt,dir,exitflag] = SIMPLEX(A,b,c)

Implementation of the two phases  simplex method for solving
min c'x  s.c. Ax = b, x >= 0

Michel Bierlaire Feb 2000
Source: D. Bertsimas and J.N. Tsitsiklis (1997)
        Introduction to linear optimization
        Athena Scientific.

Output:
 value_opt: value of the cost function when the algorithm stops
 primal_var: primal solution when the algorithm stops
 table_opt: table when the algorithms stops
 dir: if the problem is not bounded, returns a direction dir such that
       primal_var+a*dir is feasible for all a >= 0
 exitflag: 0 if an optimum solution is found
           1 if the problem is not bounded
           2 if the problem is infeasible
```

Fig. 3. MATLAB help for the SIMPLEX function.

Table 2
Paths with probability $\varepsilon/2$

1–9–10–11	1–2–3–4–10–11	1–9–10–18
1–2–3–4–10–18	3–4–10–11	6–9–10–11
3–4–10–18	6–9–10–18	4–10–11
7–6–9–10–11	4–10–18	7–6–9–10–18

Table 3
Paths with probability $(1-\varepsilon)/2$

1–9–10–18–17–16	1–2–3–4–10–18–17–16	1–9–10–11–12–13
1–2–3–4–10–11–12–13	3–4–10–18–17–16	6–9–10–18–17–16
3–4–10–11–12–13	6–9–10–11–12–13	4–10–18–17–16
7–6–9–10–18–17–16	4–10–11–12–13	7–6–9–10–11–12–13

Table 4
Probability of other paths

ε	$1-\varepsilon$
1–2–3	1–9–8
1–2	1–9–8–7
12–13	15–18

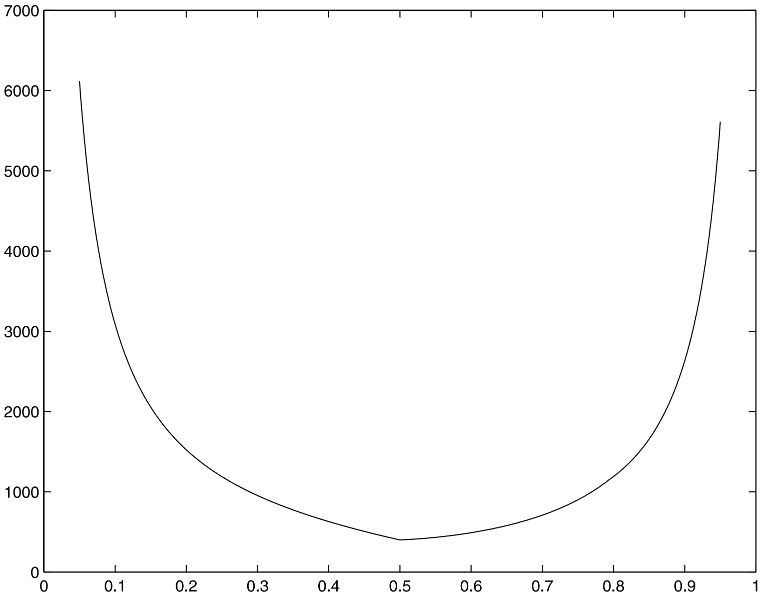

Fig. 4. Sensitivity of the TDS for various values of ε. $\hat{E} = \{9, 13\}$ and $v^*_{\hat{E}} = (400\ 200)^{\mathrm{T}}$.

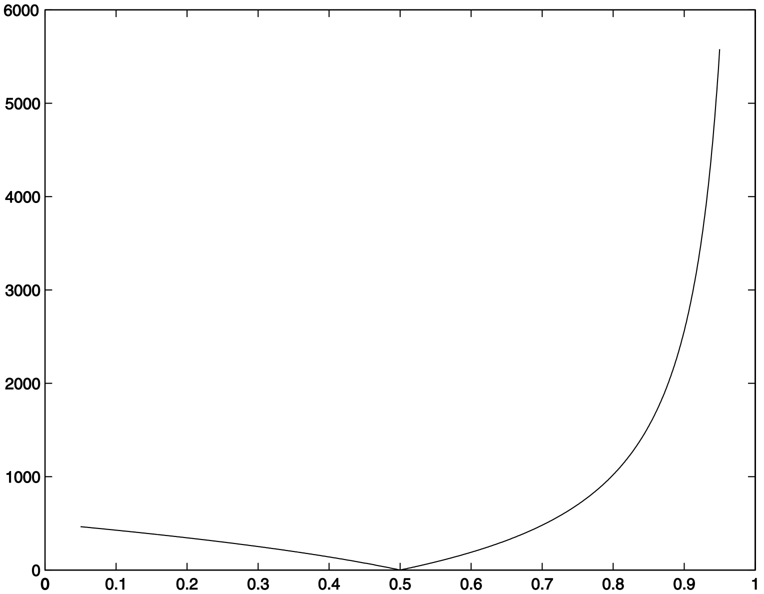

Fig. 5. Sensitivity of the TDS for various values of ε. $\hat{E} = \{9, 15\}$ and $v^*_{\hat{E}} = (400\ 50)^{\mathrm{T}}$.

probability of $(1 - \varepsilon)$. Note that the "true" route choice model used to generate the link flows is based on $\varepsilon = 0.5$.

As in Section 5.1, we consider an instance of the OD estimation problem based on $\hat{E} = \{9, 13\}$ and $v^*_{\hat{E}} = (400\ 200)^{\mathrm{T}}$. The value of the TDS for various values of ε is depicted in Fig. 4. Interestingly, we observe that the minimum value of the TDS is achieved with the "true" route choice model. Similar comments can be made with the example where $\hat{E} = \{9, 15\}$ and $v^*_{\hat{E}} = (400\ 50)^{\mathrm{T}}$ (see Fig. 5). This small example illustrates how the TDS can capture errors in the route choice assumption.

8. Conclusion

We have proposed a new measure of quality for estimated OD tables, called the *total demand scale*. Its computation involves the solution to two linear programs. The original feature of the TDS is its independence of any a priori matrix. It measures the uncertainty due to the network topology and the route choice assumptions. We emphasize that the TDS does not replace other measures of quality, but is complementary. The next step of this research is to integrate the TDS with classical variance–covariance analysis, in order to combine the strength of both approaches.

The TDS computation enables to identify OD pairs that are not observed at all and, as a consequence, helps in improving the OD table quality. There are basically two ways of doing so. First, from Theorem 1, the polyhedron is bounded if measurements are available for all links. Therefore, adding more and more measurements eventually transforms the infinite TDS into a finite one. Clearly, a clever procedure has to be derived in order to identify what measurements should be done first. Second, once unbounded OD pairs have been identified, specific and focused surveys can be conducted to increase the quality of the corresponding entries in the a priori OD table. Therefore, our approach helps to optimize the resources allocated to surveys, by identifying the nature of the necessary additional information. Finally, we have illustrated on a simple example that a high TDS may be caused by inappropriate route choice assumptions.

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